# Evaluation of Local Decision Thresholds for Distributed Detection in Wireless Sensor Networks using Multiobjective Optimization

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***Abstract*—For a distributed detection in a wireless sensor network, sensors arrive at decisions about the event of interest and send their decisions to the central fusion center. The fusion center combines the incoming sensor decisions and reaches a final decision about the absence or presence of the event. For binary sensor decisions, determination of the local sensor decision thresholds is crucial. In this paper, we evaluate the set of local sensor thresholds through multi-objective optimization where the probability of error and the total energy consumption of the network are optimized simultaneously. The optimal threshold sets are generated by using a mathematical programming Normal Boundary Intersection (NBI) method and a multi-objective evolutionary algorithm Non Dominating Sorting Genetic Algorithm (NSGA-II). Simulation results show that both NBI and NSGA-II successfully obtain a set of solutions reflecting the tradeoffs between the objectives.**

## I. Introduction

Wireless sensor networks (WSN) are typically composed of a large number of densely deployed sensor nodes that cooperatively monitor the physical or environmental conditions of an event of interest such as temperature or velocity of an object. Raw measurements or their processed versions are transmitted to a central fusion center that computes a final inference about the underlying event. WSNs are currently used in a wide range of application areas such as battlefield security, surveillance, environment or health monitoring, and disaster relief operations. In this paper, we study the detection problem where a WSN distinguishes between two or more hypotheses, such as the absence (Hypothesis 0) or presence (Hypothesis 1) of a certain event. In distributed detection, the raw measurements are first preprocessed and a quantized version of the decision statistic is sent to the fusion center. The fusion center then combines the incoming data to reach the global inference. For binary quantization and under Bayes or Neyman-Pearson (NP) performance criteria, the design of the optimal fusion rule is relatively straightforward, but the evaluation of the decision thresholds at peripheral sensors is more complicated [2]. For a given number of sensors and under the assumption of conditionally independent observations, the optimal decision rule at each sensor reduces to a likelihood ratio test (LRT) [2], [3]. The LRTs at each sensor are coupled with other sensor decisions and the fusion rule. Optimal values of the local sensor thresholds are frequently found using Person by Person Optimization (PBPO) [3]. When the number of sensors is very large, an identical decision rule for all the sensors is asymptotically optimal [4] and may be used. This simplifies the design of decision rules considerably. In this paper, we consider the case where the event has an isotropic signal emission with path loss [5]. We assume that the measurement noise across different sensors are independent and identically distributed. However, the signal strength measured at a sensor depends on the distance between the location of the sensor and the event.

In this paper, we assume ideal channels between sensors and the fusion center, (for recent work involving non-ideal channels, see [6]), i.e. the fusion center correctly receives the local sensor decisions. This requires that each sensor decision has to be transmitted with sufficient energy which is a function of the distance that it should traverse to the fusion center [7]. In a scenario, where the sensors transmit only when they detect the event, the local sensor thresholds should also be carefully designed not only to minimize the probability of error, but also to keep the energy consumption in the network below some upperbound because of the stringent energy constraints. A recent work [8] considers the design of local sensor decision rules that minimize the probability of error subject to a transmission rate constraint for each sensor. Under conditionally independent observations, a constrained minimization problem is defined and the optimal thresholds are obtained using the well known PBPO procedure.

WSN design involves simultaneous consideration of multiple conflicting objectives, such as maximizing the detection capability or minimizing the probability of error, while minimizing the transmission costs [9]. For such systems, the solution that is optimal with respect to one objective may not be optimal in terms of other objectives. For example, in distributed detection, the threshold set that minimizes the decision probability of error might require excessive amount of energy consumption. Then the designer may trade a slight increase on the probability of error for a solution with less energy consumption. Multiobjective optimization [10],[11],[12],[13] which has recently been introduced for WSN design in [9],[14], optimizes all objectives simultaneously and generates a set of solutions at once reflecting different trade-offs between the objectives. In this paper, we solve the detection problem by formulating a multiobjective optimization problem (MOP) with two objective functions, minimizing the probability of error at the fusion center (global probability of error) $P_e$ and minimizing the total network energy consumption (global energy consumption) $E_T$ as,

$$\min_{t_1,t_2,...,t_N} \{P_e(t_1,t_2,...,t_N), E_T(t_1,t_2,...,t_N)\} \tag{1}$$

$$t_{min} \leq t_i \leq t_{max} \quad i \in \{1,2,...,N\}$$

We consider a WSN consisting of $N$ sensors with parallel decision fusion. According to this model, each sensor arrives at a binary decision about the event by comparing its measurement with a threshold and the decision is sent directly to the fusion center. The variables of the MOP $\{t_1, t_2, ..., t_N\}$ are the thresholds employed at the local sensors. A solution to a MOP is said to be a Pareto optimal solution if no other solution dominates (is better than) this solution in terms of both objectives. The collection of all Pareto optimal solutions forms the Pareto optimal front (For mathematical definitions of terms *domination* and *Pareto optimality*, see [14]). To solve MOPs, minimizing the weighted sum of the objectives is a well known technique, but it has several drawbacks as mentioned in [10]. In this paper, we employ two different algorithms to solve the MOP. The Normal Boundary Intersection (NBI) method [10] converts the MOP into a number of single objective constrained subproblems. The number of subproblems determines the resolution of the Pareto optimal front and each NBI subproblem can be solved with any appropriate optimization techniques such as gradient based methods. On the other hand, if the optimization problem is not convex, such approaches may yield a local optimum instead of a global optimum. For this reason, we compared the performance of NBI with Non dominating sorting genetic algorithm-II (NSGA-II)[13] which is a state of the art multiobjective evolutionary algorithm. Initially, population of size $M$ is duplicated by crossover and mutation operations. NSGA-II is an elitist algorithm which uses a non-domination ranking approach where solutions in the duplicated population are ranked according to fronts they belong to. For instance, a solution belongs to the first front, if no other solution in the population dominates it. Similarly, the second front is composed of only the solutions that are dominated by the first front and so on.

The rest of the paper is organized as follows. In Section II, we describe the assumed WSN model and define the objective functions. In Section III, we introduce the simulation settings and then discuss the results obtained. Finally we devote Section IV to conclusions.

## II. System Model

In this section, first we introduce the wireless sensor network model assumptions. Then we define the mathematical models for both objective functions.

### A. Wireless Sensor Network Model

Figure 1 depicts an example wireless sensor network consisting of $N$ sensors, $\{s_i, i = 1, 2, .., N\}$ under parallel decision fusion architecture. The distance between $s_i$ and the fusion center and the event location are denoted as $d_{f,i}$ and $d_i$ respectively. The event location is assumed to be random and therefore $d_i$ is a random variable. Suppose that a signal that follows the power attenuation model such as an acoustic signal, is radiated from a source at position $(x, y)$ with energy $K_0$ and sensor $s_i$ is deployed at a position $(x_i, y_i)$. Then, the received event energy $(e_i)$ observed at each sensor is [5],

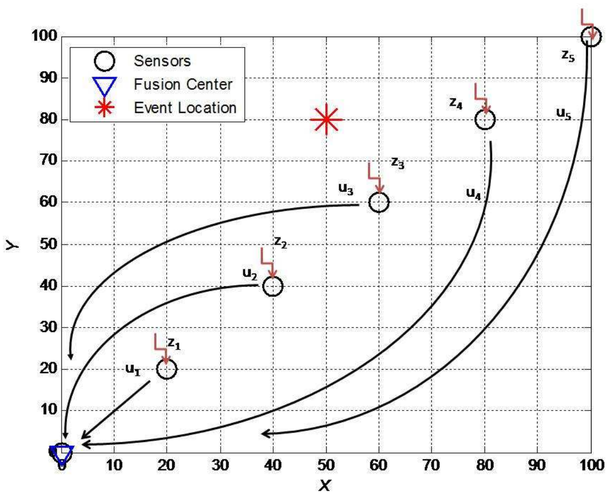


Fig. 1. Wireless Sensor Network Model

$$e_i(x_i, y_i, x, y) = \frac{K_0}{1 + a(\sqrt{(x-x_i)^2 + (y-y_i)^2})^n} \tag{2}$$

where $n$ is the signal decay exponent and $a$ is an adjustable constant. When $n = 2$, the energy of the event decays at a rate inversely proportional to the square of the distance $d_i = \sqrt{(x-x_i)^2 + (y-y_i)^2}$. Under each hypothesis, the received measurement at each sensor $(z_i)$ is expressed as,

$$z_i = n_i, \quad under \qquad H_0 \tag{3}$$

$$z_i = \sqrt{e_i(x_i, y_i, x, y)} + n_i, \quad under \qquad H_1$$

where $n_i$ is the measurement noise that follows a normal distribution and is independent at each sensor. $z_i$ then follows a normal distribution $N(0, \sigma^2)$ under $H_0$ and $N(\sqrt{e_i(x_i, y_i, x, y)}, \sigma^2)$ under $H_1$ respectively. When $z_i$ exceeds a certain threshold $t_i$, sensor $s_i$ transmits one bit decision $(u_i = 1)$ to the fusion center. Otherwise, it does not transmit anything $(u_i = 0)$. We also assume a prior probability density function (pdf) on the event location. We assume that the location of the event is uniformly distributed with joint pdf,

$$f_{x,y}(x, y) = \frac{1}{A \times B}, \qquad 0 \leq x \leq A \quad 0 \leq y \leq B \tag{4}$$

where the region of interest is an area of size $A \times B$. Then, each sensors average distance to the event location is expressed as,

$$\bar{d}_i = \int_0^A \int_0^B \sqrt{(x-x_i)^2 + (y-y_i)^2} f(x, y) dy dx \tag{5}$$

### B. Objectives

In this subsection, we derive mathematical expressions for the two objectives namely the network's global probability of error and the global energy consumption.

*1) Global Probability of Error:* Let $u_0$ be the final decision about the event at the fusion center and $P_0$ and $P_1$ be the *a priori* probabilities of $H_0$ and $H_1$ respectively. The global probability of error is then given by [3]

$$P_e = P_0 P(u_0 = 1|H_0) + P_1 P(u_0 = 0|H_1) \quad (6)$$

where $P_F = P(u_0 = 1|H_0)$ denotes the global probability of false alarm, and $P_D = P(u_0 = 1|H_1)$ denotes the global probability of detection. Under Bayesian formulation, given the vector of local sensor decisions of size $1 \times N$ ,**u** = $[u_1$ $u_2$ ... $u_N]$ and $u_i \in \{0,1\}$ the LRT that minimizes the global probability of error [3] is

$$\frac{P(\mathbf{u}|H_1)}{P(\mathbf{u}|H_0)} \gtrless_{u_0=0}^{u_0=1} \frac{P_0}{P_1} \quad (7)$$

By conditioning $P_F$ over each possible incoming vector of decisions **u** and then averaging over **u** yields the global probability of false alarm as,

$$P_F = P(u_0 = 1|H_0) = \sum_{all\ \mathbf{u}} P(u_0 = 1|\mathbf{u}) P(\mathbf{u}|H_0) \quad (8)$$

where according to the received decision vector **u**, $P(u_0 = 1|\mathbf{u})$ is either zero or one based on the fusion rule given in Eq.(7). Since the noise samples are assumed to be independent across sensors,

$$P(\mathbf{u}|H_0) = \prod_{i=1}^{N} P(u_i|H_0) \quad (9)$$

Note that,

$$P(u_i = 1|H_0) = Q(\frac{t_i}{\sigma}) \quad (10)$$

and $P(u_i = 0|H_0) = 1 - P(u_i = 1|H_0)$ where $Q(.)$ is the complementary distribution function of the Gaussian. Since the event location is random, $P(\mathbf{u}|H_1)$ can not be written directly as the product of individual decisions as given in Eq.(9). Instead, the global probability of detection needs to be first conditioned on the location of the event, and then needs to be averaged over its probability density function. For a given event location $(x, y)$, the conditional global probability of detection is,

$$P(u_0 = 1|x, y, H_1) = \sum_{all\ \mathbf{u}} P(u_0 = 1|\mathbf{u}) P(\mathbf{u}|x, y, H_1) \quad (11)$$

and since noise is independent at the sensors,

$$P(\mathbf{u}|x, y, H_1) = \prod_{i=1}^{N} P(u_i|x, y, H_1) \quad (12)$$

Then the conditional probability of detection at an individual sensor is,

$$P(u_i = 1|x, y, H_1) = Q(\frac{t_i - \sqrt{e_i(x_i, y_i, x, y)}}{\sigma}) \quad (13)$$

and $P(u_i = 0|x, y, H_1) = 1 - P(u_i = 1|x, y, H_1)$. Furthermore, the error probability of an *individual sensor* $P_{ind,i}(t_i)$ as a function of its decision threshold $t_i$ can be expressed as,

$$P_{ind,i}(t_i) = P_0 P(u_i = 1|H_0) + P_1 \int_0^A \int_0^B P(u_i = 0|x, y, H_1) dy dx \quad (14)$$

Finally, the global detection probability $P_D$, is found by averaging Eq.(11) over the probability density function of the event location as,

$$P_D = \int_0^A \int_0^B P(u_0 = 1|x, y, H_1) f(x, y) dy dx \quad (15)$$

Our first objective function, the global probability of error, is finally obtained as given in Eq. (16).

$$P_e(t_1, t_2, ..., t_N) = P_0 \sum_{all\ \mathbf{u}} P(u_0 = 1|\mathbf{u}) P(\mathbf{u}|H_0) + P_1[1 - \int_0^A \int_0^B \sum_{all\ \mathbf{u}} P(u_0 = 1|\mathbf{u}) P(\mathbf{u}|x, y, H_1) f(x, y) dy dx] \quad (16)$$

*2) Global Energy Consumption:* In this paper, we employ an energy efficient on-off keying scheme where only the sensors that detect the event transmit their decision to the fusion center. We also assume that the transmitted local decisions are delivered to the fusion center without any error. Then the energy consumption of $s_i$ for transmitting $m$ bits to the fusion center over distance $d_{f,i}$ is [7]

$$E(m, d_{f,i}) = E_{elec} \times m + \epsilon_{amp} \times m \times d_{f,i}^2 \ [Joules]. \quad (17)$$

According to this model, a sensor dissipates $E_{elec}$ = 50 nJ/bit to run the transmitter circuitry and $\epsilon_{amp} = 100 pJ/bit/m^2$ for the transmitter amplifier.

The energy consumption in the network is the total transmission energy of all single bit decisions to the fusion center, that is in Eq. (17), $m = 1$ if a sensor decides on the presence of the event ($u_i = 1$) and $m = 0$(no transmission) if it decides on the absence of the event ($u_i = 0$). An *individual sensor*'s energy consumption can be expressed as,

$$E_{ind,i}(t_i) = E(1, d_{f,i})[P(u_i = 1|H_0) P_0 + P(u_i = 1|H_1) P_1] \quad (18)$$

To transmit a given vector of decisions **u** to the fusion center requires,

$$E_C(\mathbf{u}) = \sum_{i=1}^{N} E(u_i, d_{f,i}) \ [Joules] \quad (19)$$

The energy consumption $E_T$ of the network is then found by conditioning on all possible vectors of decisions as,

$$E_T = \sum_{all\ \mathbf{u}} E_C(\mathbf{u})(P(\mathbf{u}|H_0)P(H_0) + P(\mathbf{u}|H_1)P(H_1)) \quad (20)$$

Using the relation,

$$P(\mathbf{u}|H_1) = \int_0^A \int_0^B [\prod_{i=1}^{N} P(u_i|x, y, H_1)] f(x, y) dy dx \quad (21)$$

together with Eq.(9) and Eq.(20), our second objective, the global energy consumption of the network, is obtained as defined in Eq. (22),

$$E_T(t_1, t_2, ..., t_N) = \sum_{all\ \mathbf{u}} E_C(\mathbf{u})[P_0 \prod_{i=1}^{N} P(u_i|H_0) + P_1 \int_0^A \int_0^B [\prod_{i=1}^{N} P(u_i|x, y, H_1)] f(x, y) dy dx] \quad (22)$$

## III. Simulation Results

In our simulations, we use the WSN configuration described in Section II.A. As shown in Figure 1, the proposed MOP is illustrated with deterministic sensor placements where the sensors are equidistantly placed on the $y = x$ line in the region of interest $A \times B = 100m \times 100m$. As an example, boundary or pipeline surveillance requires placing the sensors on a straight line. The fusion center is located at the origin. According to our objective functions shown in Equations (16) and (22), adding an additional sensor to the network doubles the number of possible vector of received decisions. So that the computational complexity of both objectives increase exponentially with $N$ $(O(2^N))$. For this reason, we illustrate the proposed MOP with relatively few sensors. The *a priori* probabilities for $H_0$ and $H_1$ are selected as $P_0 = 0.8$ and $P_1 = 0.2$ respectively. The parameters of the event detection model are set as: $K_0 = 10^6$ Joules, $a = 200$, and $n = 2$. The standard deviation of the measurement noise $\sigma$ is set to 1. The minimum $t_{min}$ and maximum $t_{max}$ values for the thresholds are taken as 0 and 10 respectively. For NBI, individual minimizers of each objective function and each NBI subproblem are determined by using MATLAB ©'s `fmincon` routine. For the `fmincon` routine all sensor thresholds are initialized at $t_i^0 = 8$ where $P_e^0 \approx 0.2$ and $E_T^0 \approx 0$. The resolution of the Pareto-optimal front is selected as $R_{NBI} = 10$. For NSGA-II, we use a population of size $M = 100$. Recombination and mutation probabilities are set at 0.9 and 0.1 respectively. The convergence is obtained by executing NSGA-II over $G = 200$ generations. We observed that slight changes in these parameters do not change the results significantly. For each problem, we observed the results of NSGA-II averaged over 5 different trials. NBI and NSGA-II methods are implemented via available public codes in [15] and [16] respectively.

### A. Pareto-optimal Fronts

Figure 2 shows the Pareto-optimal solutions obtained by using NBI and a single trial of the NSGA-II, where some of the solutions on the Pareto optimal front is represented with its visually objective function pair $[P_e, E_T]$. Note that adding more sensors decreases the global probability of error and increases the global energy consumption. By using NBI, the Pareto optimal solutions are evenly distributed which provides more performance alternatives for the designer. As an example, for $N = 5$ sensors, if we only consider the minimization of $P_e$, the best achievable global probability of error is $0.061$, which consumes $497.5654nJ$. Via MOP a user is provided with a set of optimal solutions to choose from. For instance, instead of selecting the minimum probability of error solution, we can select the neighboring Pareto optimal solution with objectives $[0.0619, 401.38\text{nJ}]$. Therefore, $1.5\%$ increase in the global probability of error, results in $23\%$ saving in global energy consumption. Similarly for $N = 6$ case, instead of operating at the minimum probability of error solution $[0.05, 536\text{nJ}]$, selecting the Pareto optimal solution $[0.058, 246\text{nJ}]$ yields $53.9\%$ energy saving in exchange for a $15.11\%$ increase in the global probability of error. Simulation results show that the NBI and NSGA-II yield Pareto optimal fronts that are fairly close to each other. The computational complexity of NSGA-II is $O(n_o \times M^2)$ where $n_o$ is the number of objectives and $M$ is the population size [13]. NSGA-II is computationally quite expensive since it calculates the values of both objective functions for the duplicated population and needs to repeat this process until the algorithm converges. On the other hand, for our implementation, NBI needs to solve a number of subproblems that is determined by the resolution of the Pareto-optimal front, reducing the computation time significantly as compared to NSGA-II. As an example, under $N = 5$ decision variables and given above simulation settings, one realization of NSGA-II requires $26000$ seconds, while NBI requires $4260$ seconds on a computer with a 3.2 GHz Pentium processor.

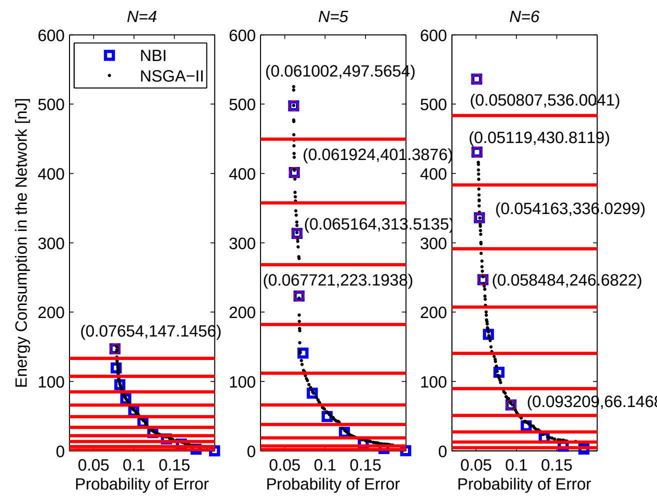


Fig. 2. Pareto optimal front via NBI and NSGA-II for $N = 4$, $N = 5$ and $N = 6$ sensors. The Square and Point markers denote the Pareto optimal solutions obtained by using NBI and NSGA-II respectively. Horizontal lines indicate the possible decision boundaries between the NBI solutions.

### B. Individual Sensor Performance

In this subsection, we analyze the performance of individual sensors based on the selected Pareto optimal solution presented in the previous subsection. First, we determine the error probability of an individual sensor $P_{ind,i}(t_i)$ as given in Eq. (14) as a function of its average distance to the event location $\bar{d}_i$. Next, we calculate an individual sensor's energy consumption $E_{ind,i}(t_i)$ as given in Eq. (18) as a function of its distance to the fusion center $d_{f,i}$. For the minimum global probability of error solutions, Figure 3 shows that the local sensor thresholds are assigned in such a way that the individual sensor error probability increases with the mean sensor distance to the event location. That is the sensors that are far away from the event location transmit rarely which make their error probability close to $P_1$. In terms of energy consumption, Figure 4 shows that energy consumption of a sensor increases with its distance to the fusion center. This is an expected result since the energy consumption of a sensor increases with the square of the distance to the fusion center. For the consecutive Pareto-optimal solutions with increased global probability of error and decreased global energy consumption, the energy consumption of the sensors that are far away from the fusion center is decreased by increasing their thresholds. Since these sensors are also relatively far from the expected event location, decreasing their transmission rate makes only a slight difference in the minimum achievable global probability of error. On the other hand, since delivering their decisions to the fusion center has much energy cost, decreasing their transmission rate provides significant savings in global energy consumption.

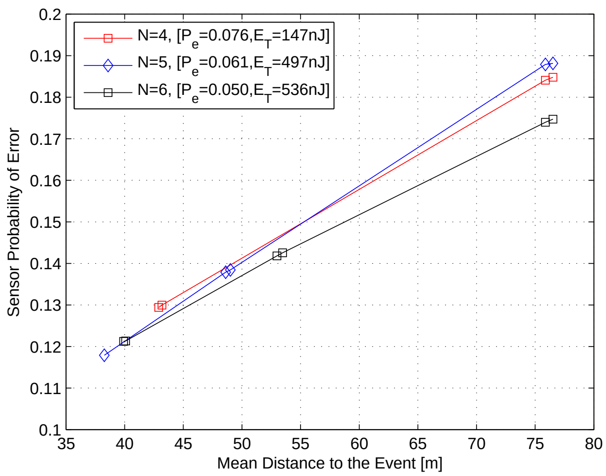


Fig. 3. Local sensor error probability vs its expected distance to the target location

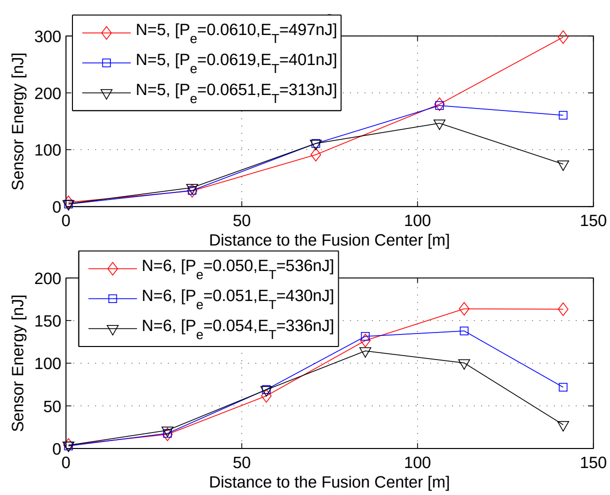


Fig. 4. Local sensor energy consumption vs its distance to the fusion center. (a)$N = 5$ sensors,(b)$N = 6$ Sensors

## IV. Conclusions

In this paper, we defined a multiobjective problem with two conflicting objectives global probability of error and global energy consumption. Each solution to this MOP problem corresponds to placing a different emphasis on the objectives. Rather than having a single solution that minimizes the global probability of error, MOP solutions provide several design alternatives which deliver significant energy savings as compared to the energy consumption of the minimum probability of error solution while increasing minimum achievable probability of error slightly. Future work comprises adapting the proposed framework to larger number of sensors if appropriate model reduction techniques are used.

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